United Kingdom
Mathematics Trust

Senior Mathematical Challenge

2 – 5 November 2020

Organised by the United Kingdom Mathematics Trust

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Candidates must be full-time students at secondary school or FE college.
England & Wales: Year 13 or below
Scotland: S6 or below
Northern Ireland: Year 14 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **90 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   All candidates start with 25 marks;
   0 marks are awarded for each question left unanswered;
   4 marks are awarded for each correct answer;
   1 mark is deducted for each incorrect answer (to discourage guessing).
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Senior Mathematical Challenge should be sent to:

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☎ 0113 365 1121 enquiry@ukmt.org.uk www.ukmt.org.uk

1. What is the value of $\dfrac{2020}{20 \times 20}$?

A 10.1 B 5.5 C 5.1 D 5.05 E 0.55

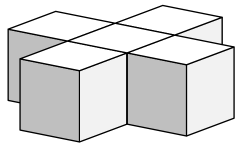
2. What is the remainder when 1234×5678 is divided by 5?

A 0 B 1 C 2 D 3 E 4

3. A shape is made from five unit cubes, as shown.

What is the surface area of the shape?

A 22 B 24 C 26 D 28 E 30



4. The numbers p, q, r and s satisfy the equations $p = 2$, $p \times q = 20$, $p \times q \times r = 202$ and $p \times q \times r \times s = 2020$.

What is the value of $p + q + r + s$?

A 32 B 32.1 C 33 D 33.1 E 34

5. What is $\sqrt{123454321}$?

A 1111111 B 111111 C 11111 D 1111 E 111

6. There are fewer than 30 students in the A-level mathematics class. One half of them play the piano, one quarter play hockey and one seventh are in the school play.

How many of the students play hockey?

A 3 B 4 C 5 D 6 E 7

7. Official UK accident statistics showed that there were 225 accidents involving teapots in one year. However, in the following year there were 47 such accidents.

What was the approximate percentage reduction in recorded accidents involving teapots from the first year to the second?

A 50% B 60% C 70% D 80% E 90%

8. What is the largest prime factor of $106^2 - 15^2$?

A 3 B 7 C 11 D 13 E 17

9. In 2018, a racing driver was allowed to use the Drag Reduction System provided that the car was within 1 second of the car ahead. Suppose that two cars were 1 second apart, each travelling at 180 km/h (in the same direction!).

How many metres apart were they?

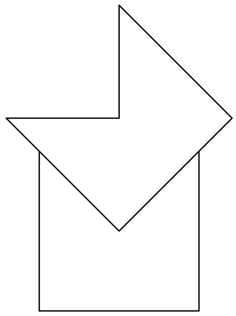
A 100 B 50 C 10 D 5 E 1

10. Six friends Pat, Qasim, Roman, Sam, Tara and Uma, stand in a line for a photograph. There are three people standing between Pat and Qasim, two between Qasim and Roman and one between Roman and Sam. Sam is not at either end of the line.

How many people are standing between Tara and Uma?

A 4 B 3 C 2 D 1 E 0

11. Two congruent pentagons are each formed by removing a right-angled isosceles triangle from a square of side-length 1. The two pentagons are then fitted together as shown.



What is the length of the perimeter of the octagon formed?

A 4 B $4 + 2\sqrt{2}$ C 5 D $6 - 2\sqrt{2}$ E 6

12. A three-piece suit consists of a jacket, a pair of trousers and a waistcoat. Two jackets and three pairs of trousers cost £380. A pair of trousers costs the same as two waistcoats.

What is the cost of a three-piece suit?

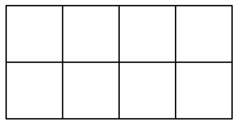
A £150 B £190 C £200 D £228
E more information needed

13. The number $16! \div 2^k$ is an odd integer. Note that $n! = 1 \times 2 \times 3 \times \cdots \times (n-1) \times n$.

What is the value of k?

A 9 B 11 C 13 D 15 E 17

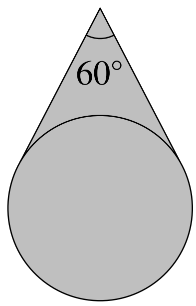
14. Diane has five identical blue disks, two identical red disks and one yellow disk. She wants to place them on the grid opposite so that each cell contains exactly one disk. The two red disks are not to be placed in cells that share a common edge.



How many different-looking completed grids can she produce?

A 96 B 108 C 144 D 180 E 216

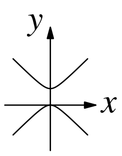
15. The shaded area shown in the diagram consists of the interior of a circle of radius 3 together with the area between the circle and two tangents to the circle. The angle between the tangents at the point where they meet is 60°.



What is the shaded area?

A $6\pi + 9\sqrt{3}$ B $15\sqrt{3}$ C 9π D $9\pi + 4\sqrt{3}$ E $6\pi + \frac{9\sqrt{3}}{4}$

16. Which diagram represents the set of all points (x, y) satisfying $y^2 - 2y = x^2 + 2x$?

A


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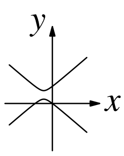

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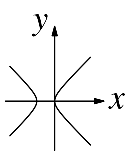
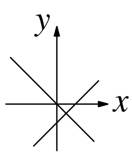
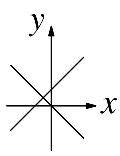


D


E


17. The positive integers m, n and p satisfy the equation $3m + \dfrac{3}{n + \frac{1}{p}} = 17.$

What is the value of p?

A 2 B 3 C 4 D 6 E 9

18. Two circles C_1 and C_2 have their centres at the point (3,4) and touch a third circle, C_3. The centre of C_3 is at the point (0,0) and its radius is 2.

What is the sum of the radii of the two circles C_1 and C_2?

A 6 B 7 C 8 D 9 E 10

19. The letters p, q, r, s and t represent different positive single-digit numbers such that $p - q = r$ and $r - s = t$.

How many different values could t have?

A 6 B 5 C 4 D 3 E 2

20. The real numbers x and y satisfy the equations $4^y = \dfrac{1}{8(\sqrt{2})^{x+2}}$ and $9^x \times 3^y = 3\sqrt{3}$.

What is the value of 5^{x+y}?

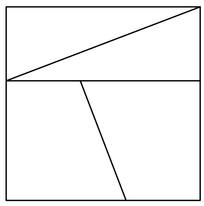
A $5\sqrt{5}$ B 5 C $\sqrt{5}$ D $\frac{1}{5}$ E $\frac{1}{\sqrt{5}}$

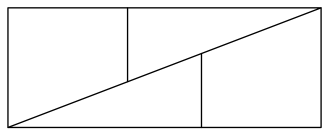
21. When written out in full, the number $(10^{2020} + 2020)^2$ has 4041 digits.

What is the sum of the digits of this 4041-digit number?

A 9 B 17 C 25 D 2048 E 4041

22. A square with perimeter 4 cm can be cut into two congruent right-angled triangles and two congruent trapezia as shown in the first diagram in such a way that the four pieces can be rearranged to form the rectangle shown in the second diagram.





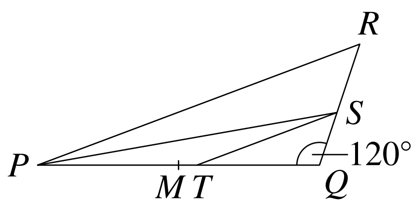
What is the perimeter, in centimetres, of this rectangle?

A $2\sqrt{5}$ B $4\sqrt{2}$ C 5 D $4\sqrt{3}$ E $3\sqrt{7}$

23. A function f satisfies $y^3 f(x) = x^3 f(y)$ and $f(3) \neq 0$. What is the value of $\dfrac{f(20) - f(2)}{f(3)}$?

A 6 B 20 C 216 D 296 E 2023

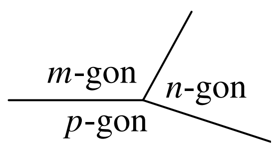
24. In the diagram shown, M is the mid-point of PQ. The line PS bisects $\angle RPQ$ and intersects RQ at S. The line ST is parallel to PR and intersects PQ at T. The length of PQ is 12 and the length of MT is 1. The angle SQT is 120°.



What is the length of SQ?

A 2 B 3 C 3.5 D 4 E 5

25. A regular m-gon, a regular n-gon and a regular p-gon share a vertex and pairwise share edges, as shown in the diagram.



What is the largest possible value of p?

A 6 B 20 C 42 D 50 E 100

